

THE MATHEMATICAL WORK-MODE AND ITS STYLES

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I. When a Theory of Literature or a Philosophy of Art are acknowledged as evident, allowing for no possible discussion, all the works included bear the term *Literature*, or *Art*. Literature, represented by different genres -drama, poetry, the novels- and Art, with its painting, sculpture and architecture, undergo continuous historical change.

More precisely, it is acknowledged as equally indisputable that works of art are written or created by someone who belongs to a particular society. No study that I know of reflects on the fact that works of literature with the characters in these works belong to a Platonic world of essence or pure forms which the artist tries to reveal through a special, *eidactic* intuition of character, although undeniably some studies refer to the beautiful and the sublime, to aesthetics in general. On the contrary, the characters who function in these works suffer, love, live, die. They are considered prototypes of the persons of fiction. /e

This last possibility concerns the creative act for which, written or composed, the author could be more than one person, or could also be unknown. This is the case with the great classical Hindu or Arab works, Byzantine art, cathedrals of the western world and romances. Furthermore, there are functions of literature and art which are social and symbolic, commercial or propagandist. Undeniably, these play a productive role.

Moreover, what is of interest here, it is now possible to discuss literary or artistic styles, distinctions between styles of author or individual and personal and general styles which an author has. For example, a distinction can be drawn between the style of Azorín and that of the generation of 98. In these areas, it is not a question of reductions, advances or accumulations, nor of paradigms, nor even of models.

The word *paradigm* is used in the broad sense of a tradition according to which the literary or artistic mode of an epoch is structured. From this meaning, a parallel can be drawn with the concept of *general style*. From

AUSEJO Elena & HORMIGON Mariano (Eds.) *Paradigms and Mathematics*. Madrid, Siglo XXI de España Editores, pp.

the many different meanings of the world *model* only two will be explained here. The first meaning concerns the thing-in-itself or thematic production. The objective being attempts to imitate, to emulate, or to admire reverentially, because, through the product, an attempt is made to reflect the canon adopted by society. The Venus de Milo is held in regard as the canon of feminine beauty; the Statue of Liberty is loved as a symbol of democracy. Secondly, in the individual sense, and especially in a middle class society, it is usual to take as a model the independent individual; the *artist*, more or less *bohemian*, to shake up the very society which gives him food and shelter, and which assigns to him precisely this function. This is an antipodean attitude which the good middle class citizen must adopt. It is a function similar to the one which assigns to philosophy that of an intellectual conscience, a substitute for religion, but, in this instance, from a more honourable social position -namely that of university professor. Conversely, the scientist is converted into a model of ethical virtue for his objective and disinterested work for the good of humanity.

Although each individual work of Titian may be distinguished from each individual work of Tintoretto (distinct individual styles) both artists share, nevertheless, the same general style, which is very different from that of Braque or Picasso. These distinct individual styles, in their diversity, shape and reflect structures and general forms, according to which, ontic and essential elements are regulated. These elements are essential to every work of art and are independent of artist and style.

Thus, in every painting or pictorial work there are ontic elements such as position, pigmentation, texture, size, shape, and amount. In every piece of music there is timbre, pitch, and harmony. In literature, in addition to the elements already mentioned, and for any critic of the symbolic imagination, ontic material reflects and shapes the great myths of the human species -the hero, the winner of the maiden, and the man of power.

These are the ontic elements, both material and formal, which have to be used with technical instruments in each art form and in each epoch to produce works of art. This work is carried out by means of regulated principles which are the characteristics of a style and, within each general style, have to be manipulated by each individual artist when he realises his work of art.

The work constructed by the individual is the result of resolving a series of problems with particular ontic and technical material, but in a style and at a time which determines such problems, for example, theme, size, and position of the work.

Think of an oil painting. The painter is commissioned to paint a work with the theme of *The Last Supper*. Clearly, for the material used - oil- and for the theme undertaken -Christian society- the ontic material typical of oil painting such as size, situation, and all the given elements, constitute the work commissioned. There is a problem here. Thirteen flat, three-dimensional figures must be created, from which one must be the principal figure, placed at the focal point which draws the most attention. The table, historically, must not be so large as to block the view of the scene. Different solutions to this problem can be studied as both the emergence or the reiteration of the problem.

In music, a standard for tonality assumes the acceptance of previous information, independent of composer and style. Different elements of sonority are essential for atonality. Yet, take one of these essentials, with its own elements of sonority, such as that of eastern tonal music, and according to the regulation of the materials through such structural forms as the sonata, individual and distinct styles are produced. Each artist creates his own characteristic, constructive, and expressive work according to the musical instrument used.

If styles are standards whereby the regulation of the ontic elements which constitute each art form are sensitively organised, then not only are rules drawn up to elaborate or to construct a concrete work; not only do the rules for games come to be composed (and, equally, those which control technical instruments), but they have to be organised and controlled each time they are used. Styles are a human production which take shape through the manner in which works of art are conceived and created. Styles consist of social concepts; those of the historical circumstances in which a style exists and in which the artist lives. These are circumstances which, in one way or another, manifest themselves as such and condition, for example, themes, acceptable types of proportion, and ways of capturing these themes.

The Byzantine conception of space is not the same as that of the Renaissance artists, of Cézanne or Miró. Spatial conceptions are not only geometrical and typological because elements such as symbols also play a role. In Byzantine art, the figure of one who donates a gift to the church cannot be the same size as that of a saint. Nor can he be positioned in front of a saint. During the Renaissance, the Virgin's triangular-shaped protector conditioned the space and the corresponding composition of the painting. Spatial conceptions such as symbols generate distinctive perspectives. Thus spatial and conceptual problems have to be resolved in each individual work.

Such an approach means that to study the style of an epoch is to understand something of its themes or contents, methods, materials,

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customs, and relationships between different productions. This simultaneously avoids two pitfalls. Firstly, it avoids talking in terms of a linear development, which implies a progression; an approach to the final work as if it were the culmination of all previous works. Secondly, it avoids incorporating elements of a modal nature such as value judgements. Such judgements assume, implicitly, the admission of a final point in historical processes, a final point which underpins the importance of the actual and, by misrepresenting this importance, gives a false impression of the significance of the individual artist.

On the other hand, learning the techniques of painting or music requires the reproduction of previous works of art with distinctive styles. How many hours must be spent at the piano in order to be able to perform (using the word *perform* in all its meaning) works such as those contained in Bach's *Notebook for Anna Magdalena*; in Schumann's *Scrapbook of Youth*; in Bartok's *Microcosmos*. These are three radically different musical worlds and the music of them needs to be practised simultaneously to reach the stage of *dominating* the technique of piano playing and, at that point, beginning to dominate the technique of composition? What were the studios of the Renaissance artists like, or what are the schools of fine art like now?

In other words, from a Theory of Literature, from a Philosophy of Art, assumptions are taken as fundamental hypotheses. The above mentioned works are human productions which reflect cultural concepts, symbols, and social aspects in general of a certain period. They are works which, in their time and as one of the functions of art, condition readers, listeners, or spectators, who then assume determined views or conceptions.

II. I believe that what has been stated here has become acceptable and accepted in the areas of Art and Literature and even, in a modified form, in some scientific disciplines. Nevertheless, when we are confronted with mathematics, this mode of thinking disappears. Any discussion of mathematics leads to confusion. There is general agreement that there are universal forms which seem to constitute an eidetic world which is radically different from the personas of fiction. These forms have to be discovered by intellectual intuition, of which we are ignorant. Or, the argument is simplistic and empirical, based on whether or not these forms are achieved through the abstraction of material by means of the mental processes of idealisation and imagination which, again, are hardly known. These arguments reflect dilemmas, such as those of Benacerraf,

between the ontological and the epistemic being, based on causal theories of knowledge and the *new* philosophies of mathematics, (mentioned in subsequent discussions) centered in existence or non-existence with a return to the mediaeval themes of universals, of the credible or the knowable. These are discussions in which, actually, there are no mathematical elements, simply allusions to elementary arithmetic or some such notions. Discussions concerning a Philosophy of Mathematics from the mathematical work mode certainly remain marginal.

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From this marginalisation, some philosophers have continued to pursue mathematics with respect to philosophy. It is considered independent of time, as an absolute that has nothing to do with its social bearing or history, because the discussion of its existence and fundamental principles is shaped in theoretical systems. There are philosophers who are opposed to this. Marx and Nietzsche both agreed that Philosophy depends on its moment in history. They accepted that it constitutes a *vision of the world* without pretensions, symbolic in character, and non-conceptual. ~~In~~ the foundation of being and, what is more, it justifies elements such as historical reason, politics, linguistics, or a certain type of state, from its position in the ideological superstructure.

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This last type of agreement has not occurred, in general, in mathematics, and discussions of mathematics is carried on with a fundamentalist and reductionist viewpoint. What is needed is for Mathematics to treat timeless objects independently of those that predicate their own essential, universal, true properties, independent of the space from which the constructive, conceptual rationality of mathematics takes all its meaning.

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This is a position which leads to confusion, among other things, between essential elements and the rules of each method. The essential elements, those which have been called ontic or substantive, are the primary data from which an object is constructed or composed. They come from what is already given, independent of style and creator, and are as much material as formal. In music, the *a priori* data are timbre, pitch and harmony. In painting, think of texture, pigmentation, volume, and form.

A musician needs ontic, sonorous materials, technical instruments and musical notes in order to compose music. But he doesn't compose music with only data previously drawn from these elements.

The essential elements for the already existing material may appear constant, uniform, and atemporal, although they have been discovered at different times. But nothing is created with only these ontic elements.

Moreover, materials can only be converted from such ontic elements with particular selections and regulations. Not everything that is sonorous is musical; much of it is merely noise.

Ontic elements, which are the necessary conditions for the construction of a work-mode are not a ~~(necessary) condition~~, nor do they have the timelessness that is assigned to them by a fundamental work-mode. They take from this assignation only those points which have to be transmitted for construction with these elements or elaborate propositions which are close to such constructions. These are points which only make sense when they have already been established in a previous style. A certain proposition only makes sense in an already determined framework. For example, theorems of existence and uniqueness are constructed theorems with an earlier character which goes back to the mathematical work-mode of the nineteenth century. This is so because, at this precise moment, previous data on existence and uniqueness split off into a different direction; it was the point of departure.

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Together, the ontic elements support the regulations which, besides categorising ontic elements which belong to previously selected materials, form a particular method. These regulatory elements, together with the technical instruments which fall within the framework, are neither necessary, nor universal, nor timeless, because they are formed as regulating principles throughout the human ~~method~~.

work.

On the other hand, different regulations can be made in addition to the element of ontic material itself, an element which changes the meaning of onticity (if the expression is permitted) from each adopted regulation.

It is necessary to bear in mind that in capturing nature, some ontic elements cannot be marked out as essentials, establishing principles now and forever. Because in the capture and transformation of ~~human~~ nature, each individual is immersed in a *Burbujas o Ambitos* which determine their own types of rationality. They condition the essential elements, just as they condition the regulations of such *Ambito*.

the species human and

These regulation changes -not only in their normative principles, but also in the technical instruments for the handling of ontic materials- can suggest other essential ontic materials. They are principal regulations which are transformed and, in that transformation, they delimit different styles. Furthermore, although the ontic materials, and even the finality of the work-mode seem the same, namely the capture of nature, the techniques with which the materials are handled, their manner of capture and the solutions to the problems which each form creates, are different.

(*) Condiciones necesarias para un hacer ; pero esas condiciones no forman la necesidad ni la atemporalidad para -
De manera q son condiciones necesarias para ; pero ellas mismas no tienen por qué ser necesarias.
No sé si queda bien reflejado ese matiz co-
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III. If the mathematical work-mode is taken to be a production and a product of the human species, inside a special *Ambito o Burbuja* such as the conceptual and, within this, the constructive, the same thing occurs artistically, literally, and philosophically as with the symbolic *Ambito* in the non-essential line of elaboration of systems and in the function, among others, of establishing possible models of nature. The result of all that I have said concerning artistic styles indicates to me that the mathematical work-model is valid, even knowing that there are work-modes which belong to different *Ambitos o Burbujas*.

Now I want to discuss styles which structure work, the procedure of mathematics. Regulated styles of ontic elements (material and formal), and using different technical instruments of the arts (sonorous, pictorial, literary), and those which are recorded as more formal (space, figure, form, interaction, continuity, transformation, symmetry, structure, and consequence), are ontic elements proven not only by the strictly conceptual and formal, but by reiterative action with a physiological base for the specific material figure and sign.

This is a style that, as one of the global conceptualisations of the mathematical work-mode, assumes the acceptance, on the one hand, of some thematic contents, among which problems can be demonstrated, apparently constantly, throughout time. On the other hand, and fundamentally, are the factors which facilitate the elaboration of these thematic contents.

Among the principles which define styles are those which are responsive to questions such as the following:

- Which classes of objects can be handled -whether it be a case of the number, the differential equation, or the function- in a figural work-mode; the whole, the ~~part~~, the structure in a global work-mode approach; the algorithm, the Turing machine, the random number, in a computational work-mode approach?

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- What types of definitions are acceptable for these objects whether they be the mechanics of Archimedes or implicit or recursive for equivalent classes?

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- What methods and types of evidence, which strategies are considered acceptable?

- Which signs, whether these be formally symbolic, schemes of figures and their categories, or a mixture of both, (and with which the constructive methods, as well as accepted evidence, are supported), require that the dependence of the mathematical work-mode with respect to writing must not be confused with language?

- Which types of discourse or expressive forms are acceptable?
- What are the problems of interest? To find the longitude of an arc of a general curve constitutes a problem of interest and of great difficulty in a strictly geometrical work-mode. The success of Torricelli with the spiral arc logarithm and the challenge of Dettonville with the cycloid are problems of no interest from the point of view of a work-mode in the ϕ style.
- What relationships are there with other disciplines, whether these be the work-mode of pure mathematics, or the language indispensable for other sciences, or essential ~~(standard)~~ for these?
- Which social production is attributable to the mathematical work-mode and, furthermore, which factors are socially relevant to the need or otherwise of general education and its psychological and social consequences, such as those attributable to *scholastic collapse*?

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IV. This characterisation of the minimum points which form mathematical styles (~~distinguishable from~~ the definition of the mathematical style by the conditions already fulfilled) has diverse consequences and needs explanation.

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On the one hand, styles are not delimited by an explicit codification of norms or principles, articulated in a the more or less organic law, approved in a Congress, or in a Parliament. Nor are they like the explicit decrees or regulations which made the French or the October Revolutions (with their associated *coups d'état*) possible. This absence of explicit crystallisation points to the difficulty in capturing, expressing and delimiting as much the concept of style as each one of its realisations. Specifically, it is a particular capture from an artistic praxis. (The notion of art is understood here in its original sense of being joined to crafts). Styles constitute critical references. They cannot be totally apprehended because not enough is given for such a grasp. The archaeological remains of a text, of a letter, or a book or the cataloguing of different methods of handling and showing these at a particular time are all standards from a subsequent view of the mathematical work-mode, and this view becomes conditioned by the standard in which it is situated. Thus it is impossible to capture the *complete* work-mode of a style of a certain epoch, as opposed to that of a different epoch. It is only possible to secure thematic content, and even then there are reservations; it becomes impossible to handle one style or another.

On the other hand, as a consequence of the characterisation of style assumed here, it must be the case, in the first place, that style reflects and

at the same time provokes the coherence -and sometimes the polemics- of those who work in this work-mode. It is a coherence from which a mathematical community of the epoch is estimated. Furthermore, it is a coherence which permits, at the same time, the conversion of the conceptual style or manner of approach and the mathematical work-mode, in ideological terms. The established community tries to impose its point of view on other individuals and tries to obtain a position of privilege in the society in which it is entrenched.

To establish this coherence, it is necessary for the society to be able to estimate what is the objective criteria of the knowledge produced. It is the style, (as a conceptual framework, which can become an ideological framework, as previously stated), which measures the normative criteria for which the work-mode of the time is conditioned, up to and including elements such as those that qualify rigour, the rigour of an epoch.

A suitable example would be the production of the Bourbaki School. This school gave impetus, and not only in content, to a style known as formal, and with it, a coherence to a mathematical community not only in the lines of creation but in those who dedicated themselves to teaching, at all levels. Of course, it also provoked polemics, and encountered reactions. Naturally, it is necessary to be precise about what was actually put forward by Bourbaki as much in the mathematical work-mode, as in the social images of this work-mode. It is also necessary to distinguish between the Bourbakist work-mode and the Bourbakist ideology. (The same holds true for figures such as Hilbert, for example, who was considered both a radical mathematician in the formal style and finitist when he is considered as a mathematical defender of the imagination in the mathematical work-mode and the radical links of mathematics with physical reality).

On this point, it is necessary to emphasise the social production of the institutional monuments of Societies, Colleges, Universities and Publishers, because they are at the heart of the crystallisation, and above all, the stabilisation of the style in which the said institutionalisation is produced. Not only are institutional monuments involved; the style allows the explanation of the production which is attributed to particular figures in an epoch, such as that of Weierstrass with respect to a style like ϵ , or to the group previously cited of Bourbaki, with the diffusion of formalism.

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Secondly, as a global conception, it allows an explanation of the conditioners which in each epoch support, with respect to ways of approaching the history of the mathematical work-mode, ways or approaches which come from each style's own perspective.

Thirdly, the institutionalisation is supported, with respect to the individual, by learning. I have been among mathematicians enough to remember the first thing that everyone did: that was to work like piano students, but with theorems and problems which had to be learnt and resolved. By doing so, we supported the incorporation of some cognitive fields already accepted as a collective, with various types of construction and demonstration and with the minimum conceptual baggage which enables sight, although not always, of the existence of certain open problems.

In learning, concepts are introduced to the student. These are precisely those of the time in which the student lives, those of the style in which he is entrenched, and not only in his epoch, but more concretely in his Faculty. With these concepts, he is conditioned to approach mathematics in a certain way; to do mathematics in a certain way.

Now, this learning doesn't imply that he is inculcated with the idea of what the mathematical work-mode is (which involves very hard work, although it also brings enormously satisfying intellectual experiences). He is limited to *discovering* an eidetic world in which everything is done.

If it is implied that this is not the necessary consequence of the premises, a false image is obtained, supported, perhaps, by a fatally historical determinism. To confuse content with the regulating framework in which it is contained, begins to make sense. It is a false image because what is important is not only domination of the content, the theorems of each theory, but also the style in which this content is manifested. When a student of music senses or feels that his work is in the field of study of one of the arts, that he is able to compose his own works and not only to *reproduce* previous works as if he were a piece of hi-fi equipment, this image is shattered. The same thing happens in the mathematical work-mode. Or, to use more fundamental images, learning to speak and later acquiring rules of grammar leads ultimately to the realisation that not every discourse is a spoken one.

The field of study of technology doesn't have as its only goal its reproduction through teaching, though this certainly allows a professional exit for the individual, and with it, the reiteration and hard communal stabilisation of a style. This field of study also makes it easy for someone who acquires knowledge of it to pose and to resolve problems, to establish theorems and to expand and modify the content of mathematics of his particular time. The posing of new problems and the modification and expansion of content all indicate that previously established knowledge has been incomplete.

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In this innovative construction, and not merely in the reproduction of the acquired, it is the creative imagination of the individual, of the mathematician (and not everyone who learns mathematics is a mathematical creator, neither is the person who is limited to reproducing professionally thematic content) which comes into play. Of course, in principle he is involved in certain areas and with certain established rules of the game in the society of the time and place in which he was born and in which he studies. Furthermore, it is a question of the approaches which this society takes towards mathematics and its related areas, as though everything in this discipline had already been done.

I said *in principle* because it is necessary to be very clearly aware of an act. Only the area of the study of style and, I repeat, not just of content, in which the mathematician works, can arrive at a breakthrough which shatters the rules of the game and facilitates a new style.

v. The previously mentioned affirmations, as opposed to a fundamental conception, derive from the conviction that the mathematical work-mode is a human production. Without assessments regarding whether it is more important than art, literature or philosophy, it simply needs to be admitted that it is a product and a production of the human species, with all the limitations that this species has. As such, this product-productions is realised in styles, in ontological-epistemological conceptual ~~standards~~; not forgetting the social or politico-economic frameworks and those that are realised as work and, with it, the corresponding production. Mathematics is produced by the individual in a mathematical community which, conditioned as it is by a style, evaluates (approvingly or not) his individual production. The community of mathematics is entrenched in a politico-economic society which, in its turn, evaluates the role of this mathematical work-mode, supporting or rejecting, in economic terms, different lines of investigation; rejecting or supporting, conceptually, different standards, different communities.

frame work;

From this point of departure, a series of problems must be established, concerning not only the more or less logical fundamentals of the mathematical work-mode, but also taking into account its function. Equally, how does the mathematical work-mode manifest itself within the *Ambitos* in which the human species is organised and, reciprocally, those which have become epistemic contributions (along with those that had contributed to the mathematical work-mode) in each *Ambito* and moment.

I have already suggested that between the *Ambito o Burbujas* in which the human species is situated, there is a Symbolic Ambito o Burbuja which has conditioned the evolution of the human species, and which continues to condition it very harshly in some countries. In this *Ambito* there is a symbolic mathematical work-mode that is reflected in numerology, astrology and in the reading of certain inviolable texts. It is a work-mode that has been an instrument to elaborate various contents and mathematical processes. It is enough to show how astrology has made possible the expansion of calculus mechanisms with the advent of quadratic or cubic interpolations in trigonometric elements. But this symbolic representation has been assumed, and I believe that there are problems for the conceptual mathematical work-mode when this work-mode seeks the construction of natural models such as spatial geometrics, which are incompatible with some of its concepts.

Together with the Symbolic Ambito there is a Conceptual one and another Technological one. Within these, there is the possibility of a historical search by means of a mathematical history right through the history of mathematics and from this, it is possible to construct a Theory or a Philosophy of mathematics (and not just a philosophy of logic or the fundamentals of mathematics in which mathematics remains marginalised) which takes into account the styles in which it is manifested as the above mentioned work-mode.

From this approach, consideration of the mathematical work-mode as the product of a certain human sub-species inside a *Burbuja* such as the Conceptual, in this case, and not that of the Symbolic or Technical, entails the search for different styles from those which have been regulating its substantive ontic components, and from those which create new objects and classifiable propositions of theorems which are grouped by theme or discipline and which, later, will be able to be constructed by deduction.

Moreover, in the search, not only in the general distinguishing marks of earlier styles, but for each of the mathematical Styles, a taxonomy of the two will be reached. This is a taxonomy which, in essence, is an ostensive definition of style which complements the information of the globally characterised marks, as previously mentioned.

The taxonomy of styles is a definition which is ostensive with that which is called the mathematical style and is one that I have proposed in my book, *Introducción al Estilo Matemático (Introduction to the Mathematical Style)*, published in 1971, although outlined in 1966. Reproduced here, this ostensive definition, (without going into the details of the characteristics of such a style, nor the historical illustrations with which these characteristics are accompanied, with the same emphasised

precision), is basically in the same expressive forms as those which upset the ontological-methodological components which underline them. In this work the styles are classified thus:

- Geometric
- Poetic
- Material
- Cartesian-Algebraic
- Indivisible
- Pure Operational
- Those of \int / ε
- Synthetic-Analytic
- Dual
- Axiomatic
- Formal
- Semi-Formal

[← I would add here the classification of:

Computational

VI. Clearly, various problems arise from a dynamic viewpoint. The classification given follows a particularly descriptive and apparently temporal line. It seems as though each style follows the previous one. In some cases this is a correct perception, but not always. Also, the simple enumeration given implies the validity of the negative. However, this forces the questions that if different styles could coexist simultaneously, how would these styles be transformed, and would these transformations entail a progression?

- With respect to coexistence, I have just indicated that in the enumeration given, different styles overlap. Affirmation of coexistence is confirmed only through a search of a historic-descriptive character. As for mathematicians, it is sufficient to suggest that they will remember their years of learning, their years of practical mathematics. I believe that everyone, throughout these years of praxis, has seen how the work-mode of mathematics and its expressive forms have changed. Further changes have occurred in the problems and themes of each discipline, the interest of the mathematical community in which they find themselves entrenched, the interests of the social communities in which mathematics

is done; in summary how the mathematical work-mode has changed and, at the same time, how different frames of reference coexist.

With regard to my experience, which has led me to the concepts I have maintained since 1966 and that I have outlined here, I have said on more than one occasion that in my years of learning, at 8.30 in the mornings I studied Projective Geometry following the ideas of Staudt, who, as Rey Pastor commented, was studied more in Spain than in Germany. I studied Projective Geometry which can, on occasions, require a set-square, and always starting from the geometric figure or schematic representation to grasp, for example, a quartic from the graphic outline as a key element for projectively classifying types of conicals. At 11.30 those same mornings I studied Projective Geometry following the ideas of Artin, where all of the previous specific processes and their related vocabulary, and all that that vocabulary supported, were absent. On those mornings, in only three hours, I went from one style, from one work-mode and conception of Projective Geometry, to another radically different one. These were two work-modes whose only link, following what I have explained, was the name of the subject.

And I could mention that I also studied geometry in the style of Monge, which began with a sentence like, *A point is two points; a straight line is two straight lines*. This was the first thing that I heard in the Mathematical Science Faculty on the 9 October 1956, and I continued with that *same* geometry, but now treated vectorially.

Apart from personal stories, the *A bas Euclide!* and the imposition of a *modern* mathematics from Congresses, such as that of Royaumont in 1959, can be regarded as a crusade of the proponents of one style against the proponents of another; styles which, clearly, had to coexist in order for the crusade to make sense. It was this crusade which, I suppose, at the time, was predominated by a different style from that supposed by its proponents. (Dieudonné little regretted afterwards that his students, on arrival at the Faculty, didn't know how to change a variable or do integration after the reform). It was predominated by a style which I classified as *computational*. It was the elimination of the geometric to the benefit of the formal which culminated in the algorithmic. It is no irony that, with a return to the visual geometric, through the iterative algorithms and the figures which they engendered, and which can be grasped visually on a computer, it was unique instrumental support, ideal for such elaboration and, it is clear, visualisation.

- With regard to the question of transformation, of its underlying mechanisms, and the question that this assumes, I have tried to give an answer in previous works such as *Los Estilos [The Styles]*. One of the

posterior to Introducción al Estilo Matemático.

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epistemological. If the epistemic inversion creates new objects, it gives rise to new fields of study, to new theories, with new methods.

In the example which follows, the new cognitive field, the new thematic content, is the Theory of groups. In this, not only are the properties attributable to the new object studied, but also the relationships which can emerge, so that a subset of the same reinforces the structure, being a sub-group. Immediately, relationships between groups become apparent and applications to other scientific fields, such as crystallography, geometry are made. It is the theory of groups which, in its time, expands and runs into other theories. It changes in part to *modern* or abstract algebra, simple Algebra, although now the name of the discipline doesn't refer only to that of solving equations, but to that of algebraic structures.

Methodologically, the epistemic inversion assumes the search for new methods from the ones which work for the new objects. Here, new methods of definition emerge (and the emergence of a definition implies that of Gergonne or the definition for abstraction or for classes of equivalence). New types of impossible definitions emerge, not fully formed, but conceived from an ontology which doesn't accept the previous existence of collectivities to those which characterise implicitly or those which establish the relationship of equivalence.

But the methodological question also arises of ensuring that the objects which have been allowed in and which seem to fulfill the conditions of the group or system to which they belong, effectively exist and fulfill those conditions, and demonstrations of existence and unity arise as a condition almost before the continuation of the work. The epistemic inversion, in this example, in order to provoke a new style, is forced to pass from a Figural Work-Mode to a Global Work-Mode.

VII. I could continue with the intention of explaining other mechanisms of the transformation of styles, the production of distinct work-modes -*Figural*, *Global* and *Computational*- and their relationship with the coexistence of these and the ontological planes which come together. But allow me, in conclusion, to use the words of Barrow, who explains the possible existence of different styles, and, at the same time, outlines some of their differences:

I could continue with an apagogic discussion, but what's the point?

VIII. BIBLIOGRAPHY

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